**Philosophical Method Day 5: Logical Connectives and Translation**

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| **Content:**  1. Atomic Sentences (10 minutes)  2. Logical Connectives (40 minutes) | **Method:**  1. Lecture  2. Interactive lecture |

***Instructor’s Introduction***: This lesson will introduce students to the symbols used in formal logic. It develops the knowledge and skills students will need in order to translate English arguments into symbolic form (propositional logic) so that they can use formal methods (such as truth tables) to precisely evaluate their validity.

# *Goals and Key Concepts*

1. Students should understand what atomic sentences are and how they are symbolized.
2. Students should understand what logical connectives are and what the truth tables are for conjunction, negation, disjunction, material conditional, and material biconditional.
3. Students should understand why the truth tales for these connectives are as they are.
4. Key concepts: **atomic sentence, logical connective, conjunction, disjunction, negation, material conditional, material biconditional**

**1. Atomic Sentences**

Atomic sentences are statements which express one proposition (one simple claim about the world). Some examples:

Today is Thursday.

The atomic number of hydrogen is one.

It is raining outside right now.

The following are not propositions:

The atomic number of hydrogen is one, and the atomic number of helium is two.

Are you hungry?

Tomorrow is not a weekday.

We can symbolize atomic sentences by using a capital letter. Traditionally, logicians like to start with P and then go on alphabetically from there, but we can also use letters that reflect the content of the sentence, e.g., H for “the atomic number of hydrogen is 1,” or T for “today is Thursday.”

**2. Connectives**

We can make more complex statements by connecting atomic sentences using what we call connectives. We’ve already seen a couple of connectives. For example, “and” connected “the atomic number of hydrogen is one” and “the atomic number of helium is two” in the first argument. “And” is one way in English of using the connective known as conjunction. Other stylistic variants of conjunction include “but,” and “although.” As we saw above, conjunction connects two sentences by saying that they both are true. From now on, to make things simpler, we’ll symbolize connectives as well as atomic sentences.

**Conjunction**

Symbol: ∧

Example: P ∧ Q

Stylistic variants: and, but, although, in addition to

The truth table for conjunction is the following:

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P ∧ Q** |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

What are some other connectives that come up commonly in English?

**Disjunction**

Symbol: ∨

Stylistic variants: or, either or, unless

Example : P ∨ Q

Start filling in the truth table for disjunction as a class, working from the bottom up. When you get to the top row, there’ll likely be a disagreement about whether P ∨ Q should be true when both disjuncts are true. That’s because in English there are two senses of disjunction that we use: inclusive and exclusive. The inclusive sense of P or Q means that at least one of (and possibly both of) P and Q is true; the exclusive sense of P or Q means that exactly one of P and Q is true. In logic, we have decided on a convention: we translate a disjunction as inclusive except in cases where it just doesn’t make any sense to interpret it that way (for example, “I’ll roll a 7 or 11 on this roll”—you can’t roll both.) There are a lot of reasons for defaulting to the inclusive sense. One is that it’s the most charitable for the author of whatever passage we’re translating into symbols (because it rules out the least possibilities). More importantly, we need to pick one thing for the symbol (∨) to mean, and we can add on more to our translation to capture the exclusive sense of a disjunction but if “∨” were to mean exclusive disjunction then it would be difficult to easily write the inclusive sense. (Below we’ll see how to write an exclusive disjunction.)

The truth table for disjunction is the following:

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P ∨ Q** |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

**Negation**

Negation is another connective we’ve already seen. In English we typically denote it by using “not,’ though more formally it’s “it is not the case that” or we can use certain prefixes such as “un-,” “non-,” etc. It is a strange connective in that it operates on only one atomic sentence rather than connecting two (or more) together. However, technically a connective operates on one or more atomic sentences to create a more complex sentence. “It is not the case that” accomplishes just that!

Symbol: ¬

Stylistic variants: it is not the case that, not, un-, non-

Example: ¬P

The truth table for negation is the following:

|  |  |
| --- | --- |
| **P** | **¬P** |
| T | F |
| F | T |

Going back to disjunction, we can now write an exclusive disjunction as (P ∨ Q) ∧ ¬(P ∧ Q). Translating back to English, that says “P or Q, but not both P and Q.”

**Material Conditional**

Symbol: →

Stylistic variants: if…then, given that, only if

Example: P → Q

The truth table for material conditional is the following:

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P → Q** |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

It can be difficult to intuitively work out what the truth table should be for “if P then Q,” particularly the last two rows. The material conditional sort of says whenever P is the case then Q is as well. So what do we do if P isn’t the case? Again, there are actually multiple senses of the conditional that we use in English. And again, we’ve decided to pick one to correspond to the symbol (which we then term the “material” conditional as opposed to other types of conditionals. Top make the following discussion easier, explain that for P → Q we call P the antecedent and Q the consequent. Here’s an example you can use to motivate the truth table for the material conditional:

Consider: if x > 4 then x > 2 or, symbolically, (x > 4) → (x > 2)

Ask students if this material conditional is true. Presumably they’ll say yes. Ask if it’s always true or just sometimes true. Again, presumably they’ll all assent that it’s always true. Now work through the possible cases (like the rows of the truth table). You can use the following examples:

1. X = 5

Then both the antecedent and the consequent are true.

1. X = 3

Then the antecedent is false but the consequent is true.

1. X = 1

Then both the antecedent and the consequent are true.

Notice that we can’t find a value for x that makes the antecedent true but the consequent false. That’s because this material conditional is always true. In other words, we can’t create a situation like row two of the truth table for a material conditional. But we said that this material conditional is always true. And that includes the cases when x = 3 and x = 1 as well as when x > 4. In other words, it seems like rows three and four of the truth table should be true!

**Material Biconditional**

Symbol: ↔

Stylistic variants: if and only if, just in the case that

Example: P ↔ Q

The truth table for the material biconditional is the following:

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P ↔ Q** |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Intuitively, P if and only if Q means that P and Q should always possess the same truth value, which is reflected in the truth table.

We could get by without using a separate symbol for the material biconditional. Instead of P ↔ Q we could always write (P → Q) ∧ (Q → P), sort of how we can write the longer statement for the exclusive disjunction. However, that’s awfully long to write out and the material biconditional comes up far more often in arguments than the exclusive disjunction. In the end, we usually use these five connectives (negation, conjunction, disjunction, material conditional, and material biconditional) because that gives us a good balance between simplicity and brevity. We could go down to two of these connectives (negation plus any one of conjunction, disjunction, or material conditional), but then we’d find ourselves writing much longer translations on average, which would be time consuming. (Actually, we could go all the way down to just one connective, though none of these five standard connectives would do the job—we’d have to use either something called joint denial or one called the Sheffer stroke. Either way, we’d get even longer, messier translations.) On the other hand, we could use more connectives than these standard five (for example, ∨ for exclusive disjunction, and there are other possible connectives), but then we’d have to memorize more connectives, more symbols, and more truth tables, and these wouldn’t be worth it given how rarely we’d want to use these extra symbols. So the standard five connectives give us a nice balance and allow us to translate any truth-functional\* connectives that come up in English, even if sometimes we have to use several connectives.

\* Yes, we are adding an important technical qualification here, but at the level of this course, it’s unimportant. Suffice it to say that the somewhat oversimplified idea here is that we’re limiting ourselves to any connective for which you can make a truth table.

ASSIGNMENT: The testing validity using truth tables worksheet.